Model of Bid-Ask Prices

# With No Counterparty Risk

I adapt the model from Biais (1993) for centralized trading where all market participants can observe the bids, asks and market orders of their market participants. The model is a sequential game. In the first stage, two competing dealers (liquidity providers) receive a random inventory position in the risky asset between [-R, R]. In stage 2, the dealers set their bid and ask prices. In the next stage there is a liquidity shock with probability . If there is a liquidity shock, a liquidity trader (liquidity demander) receives an inventory of quantity with probability (or market sell order of size with the same probability). He then decides the size of the optimal market order, which is executed at the best bid or ask price posted by the dealers. If there is no liquidity shock, no trade takes place. In the final stage of the game, the price of the risky asset is realized and players receive their utility.

I assume that the two dealers are identical except for their inventory positions. All market participants have constant relative risk aversion utility . If there is a liquidity shock, the liquidity trader observes the market prices and selects the quantity (size) of the market order. The final price of the risky asset is , where .

# The optimal size of the market order

I analyze the case where the liquidity trader receives a liquidity shock (the case where the liquidity trader receives a shock will be analogous). At the end of the game, once the asset price is realized, the trader receives wealth:

Where is the net position of the trader in the risky asset at the end of the game and is the cash from selling units of the risky asset at the best (highest) bid price. The liquidity trader will maximize his expected utility . The optimal quantity is:

Where I have used the fact that

# The optimal bid and ask quotes for the dealer

For simplicity, I analyze the case of dealer 1, competing over a market buy order. If he has the best price, he receives the order flow and has wealth:

Where: is the bid price set by dealer 1, is the size of the market order and is his random inventory position (a number between ). If he does not have the best price, he receives:

## Reservation Quotes

The dealer is indifferent between trading and not trading when the expected utility from both actions are the same. This happens when:

Where the price is subscripted with to emphasize it is the reservation price. A similar analysis holds for the ask price:

And in general for dealer , the reservation prices are:

## Optimal quotes

Assume that the competing dealers do not observe each other’s inventory levels, but both assume that the others inventory is drawn uniformly from . I analyze the case of the optimal bid quote for dealer 1. By increasing his bid quote, he increases his probability of winning the order flow, but he must balance this against the fact that he pays more for each unit acquired. He would not like to increase his quote beyond his reservation price. The optimal bid quote is:

Similarly, the optimal ask quote is:

And in general, the optimal bid and ask quotes for dealer are:

The observed bid ask spread is:

NB: Under competition with many dealers, the second term on the RHS of the above equation is , which approaches as , and the bid ask quotes become the reservation quotes.

# Optimal quotes under counterparty risk

Under the scenario where there is counterparty risk, if the counterparty defaults the value of the asset is impaired (the holder of the non-defaulting leg no longer receives expected cash flows). However, for the defaulter, the value of the asset is enhanced (as he no longer needs to make payments). I model this as an additional shock to the realized value of the asset: . The analysis remains essentially the same, but the optimal market order size, reservation prices, and optimal quotes now become: